AC electrical conductivity of metal

$\vec{E}(t) = \vec{E}(\omega) e^{-i\omega t}$ Ext field. assume uniform across sample

However electron may not be able to move as fast as the electric field switches. Therefore the response must be frequency dependent.

Recall, $\frac{d\vec{p}}{dt} = -\frac{\vec{p}}{\tau} - eE$ Seek an oscillatory solution: $\vec{p}(t) = \vec{p}(\omega) e^{i\omega t}$

$\Rightarrow -i\omega\, p(\omega) e^{i\omega t} = -\frac{p(\omega) e^{i\omega t}}{\tau} - eE(\omega) e^{i\omega t}$

$\Rightarrow i\omega\, p(\omega) = -\frac{p(\omega)}{\tau} - eE(\omega)$ $\Rightarrow p(\omega) = -\frac{eE(\omega)}{\frac{1}{\tau} - i\omega} = \frac{-e\tau E(\omega)}{1 - i\omega\tau}$

$\Rightarrow j = -\frac{ne\,p(\omega)}{m} = \frac{\tau e^2 n E(\omega)}{m(1 - i\omega\tau)} = \frac{(ne^2\tau/m)E(\omega)}{1 - i\omega\tau} = \frac{\sigma_{DC} E(\omega)}{1 - i\omega\tau} = \sigma(\omega) E(\omega)$

Since we considered $\vec{E}(\omega)$ to be uniform we can use $\sigma(\omega)$ for long wavelengths $\lambda >> \tau$.

∴ $\sigma(\omega)$ implies movement of charge under influence of long λ.

$p(t) = p(\omega) e^{-i\omega t} \Rightarrow j(t) = j(\omega) e^{-i\omega t}$

∴ Eqn of continuity: $\nabla \cdot j = -\frac{\partial \rho}{\partial t}$ $\Rightarrow \rho(x,t) = \rho(x,\omega) e^{-i\omega t}$

$\Rightarrow \nabla \cdot j(\vec{r},\omega) = i\omega \rho(\vec{r},\omega)$

Recall $j(\bar{r},\omega) = \sigma(\omega)\, E(\bar{r},\omega)$

$$\therefore \nabla \cdot \left(\sigma(\omega)\, E(r\omega)\right) = i\omega\, \rho(\bar{r},\omega)$$

$$\Rightarrow \sigma(\omega)\, \nabla \cdot E(r\omega) = i\omega\, \rho(\bar{r},\omega)$$

Gauss Law : $\nabla \cdot E(\bar{r},\omega) = 4\pi \rho(\bar{r},\omega)$

$$\Rightarrow \sigma(\omega)\, 4\pi \rho(\bar{r},\omega) = i\omega\, \rho(\bar{r},\omega)$$

$$\Rightarrow \rho(\bar{r},\omega)\left(4\pi\sigma(\omega) - i\omega\right) = 0$$

$$\Rightarrow i\omega = 4\pi\sigma(\omega) \Rightarrow i\omega = \frac{4\pi\, ne^2\tau/m}{1 - i\omega\tau}$$

$$\Rightarrow i\omega = 4\pi\, ne^2(\tau/m)\,\frac{(1 + i\omega\tau)}{1+\omega^2\tau^2}$$

$$\Rightarrow \underbrace{\frac{4\pi\, ne^2(\tau/m)}{1+\omega^2\tau^2} = 0}_{A} \text{ and } \omega = \frac{4\pi\omega\, ne^2(\tau^2/m)}{1+\omega^2\tau^2} \rightarrow B$$

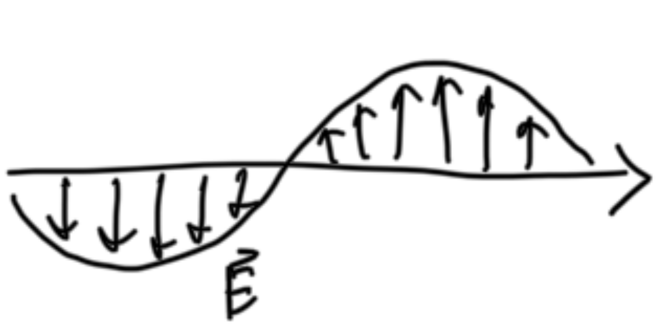
A $\rightarrow \tau \rightarrow \infty$: No collission ; collective oscillation.
(since τ cannot be zero)

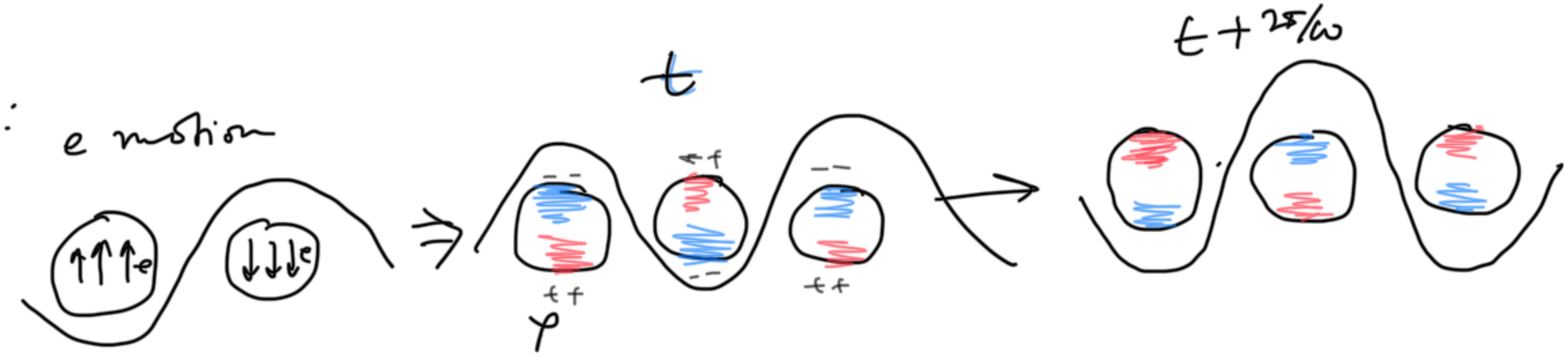
At lim $\tau\rightarrow\infty$ B $\rightarrow$ $$\omega = \frac{4\pi\omega\, ne^2/m}{\omega^2(1 + 1/\omega^2\tau^2)} \approx \frac{4\pi\omega\, ne^2/m}{\omega^2}\left(1 - \frac{1}{\omega^2\tau^2}\right)$$

$$\Rightarrow \omega^2 \approx \frac{4\pi\, ne^2}{m} \Rightarrow \omega_p \approx \sqrt{\frac{4\pi ne^2}{m}} \longrightarrow \text{plasma frequency.}$$

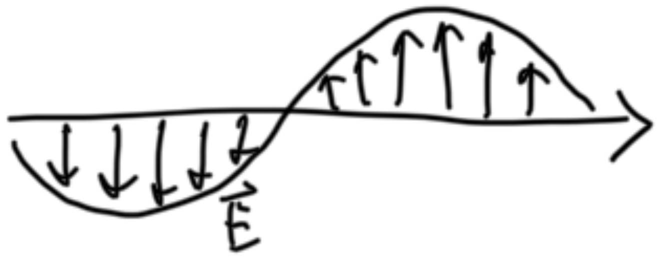
Therefore at ω_p: collective oscillation across the entire system ($\tau\rightarrow\infty$)

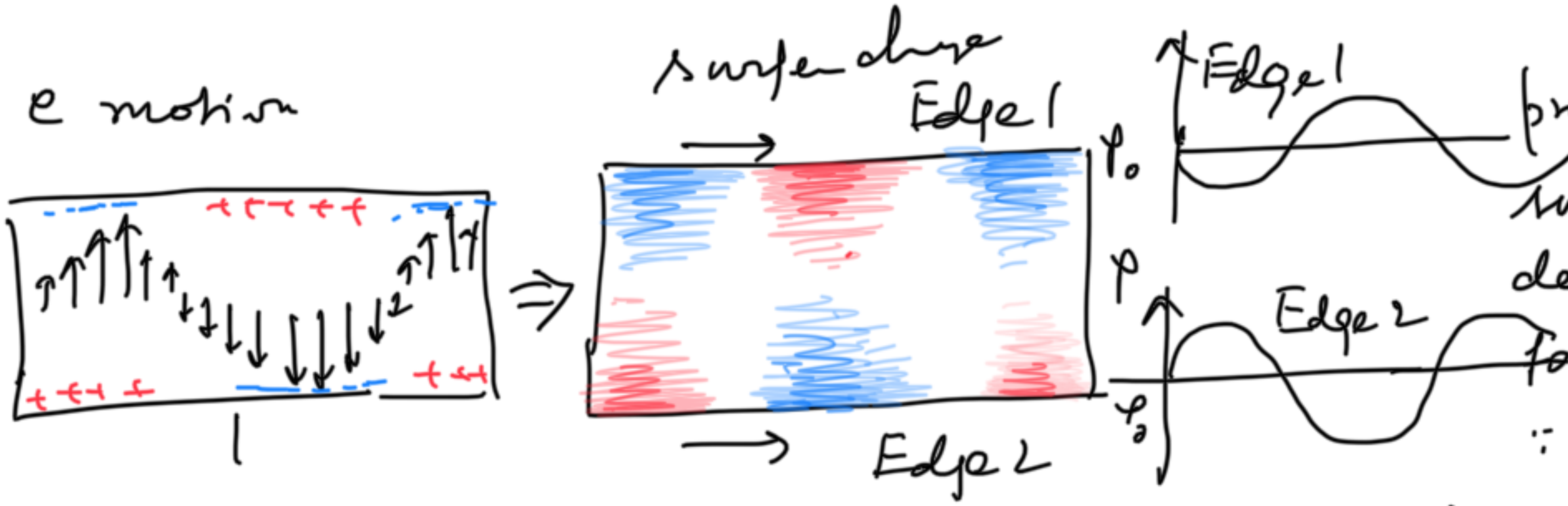
In Nanoparticles:





Extended system:





propagating surface charge density wave following $\vec{E}$

$\because E = \frac{\sigma}{\epsilon}$

$\sigma \to$ surface charge density.

Inside metal: $\vec{E}$ non zero for AC field since never reaches steady st.

φ induced is zero since charges move and accumulate only at surfaces ∞ away.

$\therefore$ MW Eq^n. (in matter) (Gaussian convention)

$\nabla \cdot E = 0$; $\nabla \cdot H = 0$; $\nabla \times E = -\frac{1}{c}\frac{\partial H}{\partial t}$; $\nabla \times H = \frac{4\pi}{c} j_{free} + \frac{1}{c}\frac{\partial E}{\partial t}$

$P = 0$ $\quad M = 0$

$(\nabla \cdot D = 4\pi\rho_{free})$ $\quad (\nabla \cdot B = 0)$

$D = E + 4\pi P$ $\quad B = H + 4\pi M$

with $P = \chi E$

$D = \varepsilon E$

$\varepsilon = 1 + 4\pi\chi$

ε → permitivity; χ → susceptibility

$G \to SI$
$4\pi \to 1/\varepsilon_0$
$4\pi \to \mu_0$

Seek oscillatory sol^n: $f(\omega)e^{-i\omega t} = f_\omega(t)$

$$\nabla \times (\nabla \times E) = \frac{i\omega}{c}\frac{\partial}{\partial t}\Big(\frac{4\pi}{c}\underbrace{\sigma(\omega)E(\omega)}_{j(\omega)} - \frac{1}{c}i\omega E(\omega)\Big)$$

$$\Rightarrow -\nabla^2 E(\omega) = \frac{\omega^2}{c^2}\Big(i\frac{4\pi\sigma}{\omega} + 1\Big)E(\omega)$$

$$\Rightarrow -\nabla^2 E(\omega) = \frac{\omega^2}{c^2}\varepsilon(\omega)\,E(\omega) \quad ; \quad \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

Complex dielectric constant: $\varepsilon(\omega) = 1 + i\frac{4\pi\sigma}{\omega}$

Recall, $\omega_p^2 = \frac{4\pi n e^2}{m}$; $\sigma_{AC} = \frac{ne^2\tau/m}{1 - i\omega\tau} \sim i\frac{ne^2}{\omega m}$ if $\omega\tau \gg 1$

ie $\tau \gg \frac{1}{\nu} \Rightarrow \nu$ high (High frequency limit.)

$$\therefore \varepsilon(\omega) = 1 + i\frac{4\pi}{\omega}\, i\frac{ne^2}{\omega m} = 1 - \frac{4\pi ne^2}{\omega^2 m} = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\therefore \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \Rightarrow \varepsilon(\omega) \to 0 \text{ if } \omega \to \omega_p$$

consistent with $D = 0$ since no free charge (induced)

$\therefore$ At high freq limit ($\omega\tau \gg 1$):

$\therefore$ $\omega < \omega_p \to \varepsilon(\omega) < 0 \to \sqrt{\varepsilon(\omega)}$ imaginary $\to$ E decay exponentially in space.

$\Rightarrow$ No radiation propagates.

all energy lost to motion of electron.

$\omega > \omega_p \to \varepsilon(\omega) > 0 \to \sqrt{\varepsilon} \to$ real $\to$ radiation can propagate

$\Rightarrow$ metal becomes transparent.

However owing to the approximate use of σ_{AC} $\left(\sim \frac{ine}{\omega m}\right)$ $\omega > \omega_p \to$ translucent

some loss due to the small real part of σ.